**4.9 Matrix Transformations from** R*n* to R*m*

A *matrix transformation* T: R*n* R*m* is a mapping of the form

**T () = A** ,

for all vectors in R*n*and an *m* x *n* matrix A.

The matrix transformations are precisely the *linear transformations* from R*n* to R*m*, that is, the transformations with the linearity properties

We will use these two properties as the starting point for defining more general linear transformations.

**Remark:**

It is important to note that a linear transformation is a special kind of function.

The input and output are both vectors.

If we denote the output vector **T ()** by we can write

**= A**

**Example 1:**

Consider the letter L in figure, made up of the vectors (1, 0) or and (0, 2) or , show that the effect of the linear transformation

**T ()** =

on this letter, describe the transformation.

**Solution:** As **T () = A**

T = =

T= =

The effect of transformation on the L is rotated through an angle of 900 in the anticlockwise direction.

**Work to do:**

Q1. Consider the matrices A= , B = ,

C = , D = ,

E = , F =

Show the effect of each of these matrices on L shape in example 1 and describe each of the transformation in words.

Solution: As, **T () = A**

T = =

T= =

For **T () = C**

T = =

T= =

.

.

.

**Types of linear Transformations**

There are two types of linear transformations (defining from R2 to R2):

1. Euclidean Transformation
2. Affine Transformation

**1 – Euclidean Transformations**

A Euclidean Transformation is a transformation T: defined by

**T(*x*) = A + ,**

Where A is an orthogonal 2 x 2 matrix and . These types of transformations always preserve distance/shape.

An orthogonal matrix holds the property **AAT= 1 or AT= A-1**

**2 – Affine Transformations**

An affine transformation is a transformation T: R2 to R2 define by

**T(x) =A + ,**

where A is a 2x2 invertible matrix and .

**Remarks:**

1.Every orthogonal matrix is invertible but an invertible matrix may or may not be orthogonal.

1. Euclidean geometry is a subset of affine geometry or Affine transformations are the generalization of Euclidean transformation.

**Types of Euclidean transformation:**

1. Translation,
2. Reflection,
3. Rotation.

**Types of Affine transformation:**

1. Scaling
2. Stretching
3. Shearing
4. **Translation**

Translation is a transformation from (R2 to R2) or (R3 to R3) defined as:

**T () = + ,**

where matrix A is the identity matrix.

**Example 2: (Translation of a triangle)**

Let A = (-2, -2), B = (2, -2), C = (0, 2) form a triangle. Find the translated triangle with vector = .

**Solution:** As the transformation of translation is

**T () = +** .

**For point A:** D = T (A) = + =

**For point B:** E = T (B) = + =

**For point C:** F = T (C) = + =

**Example -3: (Translation of a line)**

For a line 3x - 4y = 2, find the equation of line translated through vector = (2, 3).

**Solution:** The transformation of translation is:

**T () = +**

**So = +**

**Or =**

Which implies = x + 2 and = y + 3.

Then, *x* = - 2 and y = - 3.

Put these in our given equation of line that is

3(– 2) – 4(– 3) = 2

3 – 4 = -4 is the required translated line.

To draw original line 3x – 4y = 2

put ***x* = 0** implies y = -1/2, so **A** (0, -1/2) is a point on this line.

Similarly ***x* =1** implies y = ¼ and **B** = (1, 1/4) is another point on it.

In the same manner to draw the Translated line 3 - 4= -4

Putting ***x* = 0** gives= 1 and **C** = (0, 1).

Putting ***x* =1** provides = 7/4 and **D**= (1, 7/4).

**Original line**

**Translated line**

**Example -4: (Translation of circle)**

Let (x – 4) 2 + (y - 3)2 = 9 be a circle. Find the equation of the translated circle using vector (2, 3).

Note: As equation of circle: (x – a) 2 + (y – b) 2 = with Centre = (a, b) and Radius = r. While x2 + y2 = r2 is circle with Center = (0, 0) and Radius = r .

Solution: The transformation of translation is

**T () = +**

**= + =**

= x + 2 then *x* = - 2 and = y + 3 then y = - 3

Putting these equations in the equation of circle

(x - 4)2 + (y - 3)2 = 9

(-2 - 4)2 + (- 3 - 3)2 = 9

(- 6)2 + (– 6)2 = 9

Hence, Original circle is (x - 4)2 + (y - 3)2 = 9 with Center = (4, 3), Radius = 3.

While Translated circle is (**-** 6)2 + (- 6)2 = 9 with Center = (6, 6), Radius = 3

Note: As we said earlier that **Euclidean transformation**s are distance/shape preserving. So in all above examples we can see that translation transformation being a Euclidean transformation preserves the shape of each object and just translated or moved the object.

**Work to do:**

Q1.Let A = (3, 4), B = (3, 2), C = (6, 2) and D = (6, 4) form a rectangle. Find its translation thorough vector (3, 5) and verify your translated rectangle from the figure below.

Translated rectangle

Original rectangle